Optimisation and game theory approaches for allocation in LCAs

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Abstract

In LCA practice, environmental burdens are largely allocated using technical and scientific approaches. However, it can be shown that the allocation rule can also be the result of optimising goals, for example maximising the benefit of a product system. The benefit can contain ecological objectives. Furthermore, it can be shown that with the assumption of stability, efficiency and fairness, it is also expedient to apply approaches from cooperation games theory.

Keywords: Joint Production, Allocation, Optimisation, Game Theory, Shapley-Value

1. Introduction

The problem of allocation has been a topic of discussion ever since the start of Life Cycle Assessment (LCA) [1-3]. It can substantially influence the results of LCA and define the significance in product life cycles of e.g. by-products or recycling. The aim is to allocate environmental impacts to the various products in line with their actual causes. However, the problem occurs not only in classic by-products. Allocation is also necessary when transporting goods or dividing environmental burdens between the various uses in recycled products.

Boguski et al. point out that there are many methods of allocation: "*Science does not dictate a method of coproduct allocation."* [4] However, they preferred a technical or scientific method: "*Allocation on the basis of economic value is generally discouraged because the LCI methodology is based on the measurement of physical parameters, and economic value is not a physical parameter."* [4] This position is also to be found in ISO 14040 [5]. It is adopted by recent authors too [6].

Co-products in LCAs are mainly allocated on the basis of mass. However, heat of reaction or stoichiometry are also used. Boguski et al.: *If the molecules of one raw material in a chemical reaction contribute solely to one of several coproducts, then allocation of that raw material to only one coproduct may make more sense than coproduct allocation on a total-mass basis.*" [4]

By contrast, the use of economic parameters is felt to be arbitrary [6]. The question as to whether a scientific or an economic allocation should be applied is therefore almost an ideological issue, but both sides have their advocates [6-8]. Consequently allocations repeatedly account for a lack of transparency in LCAs. Attempts are then made to resolve the subsequent uncertainties in the results, e.g. through scenarios or computation variations.

Yet there are good reasons for both approaches, but they depend quite crucially on the respective situation. If they are applied wrongly, both approaches can lead to arbitrary results.

2. Causal allocation

The stoichiometric allocation mentioned is the classic example of a scientific approach. It is stringently causal and enquires where the atoms or molecules from the input materials remain. The significance of this approach for environmental sciences can be explained in historical terms. In the early days of LCA, elimination of wastes was an important topic. Many LCAs were drawn up for packaging materials [9-11]. For instance, the problem arose as to how the emissions of a waste incineration plant could be allocated to the waste.

The allocation problem here is expanded, as it were. The emissions are not to be distributed to co-products as an output, but instead divided between a number of inputs (wastes). In general it can be said that in the case of an allocation an environmental burden has to be distributed among various benefits. These can be products on the output side of a production process, or services or inputs such as e.g. wastes. It is always crucial here to know what the benefit of the process is, and what the economic effort (cost) or ecological effort (waste, emissions) is.

The problem of the waste incineration plant is that purely segregated waste is practically never incinerated. It is always mixed fractions that are burned. These mixed fractions all have different chemical compositions. That is why it is obvious that different products of incineration result from them. For instance if mercury (found in exhaust gases) is incinerated, it must be filtered out at great cost. If the mercury is allocated to waste fractions on the basis of mass or price, other materials to be disposed of which in chemical terms do not contain any mercury at all could be debited with this burden. This allocation appears to be purely arbitrary. Instead it would be desirable that only those materials in the waste which actually contain mercury in the corresponding quantities be debited with mercury emissions. This would be a clear case of "the polluter pays" allocation on the basis of the causality principle.

The problem lies in the practical implementation. There are seldom detailed data about what chemical elements are contained in the various waste fractions and what their influence on the incineration and emissions is. There are admittedly comprehensive models that allow this kind of question to be calculated [12], but generally only mixed values for common domestic waste are stated [13].

3. Allocation on the basis of prices

However, the scientific method with stoichiometric allocation can become arbitrary too. One good example of this is chlorine-alkali electrolysis, which consumes large quantities of electrical energy and is linked with emissions. Altogether three chemical products result: chlorine, caustic soda and hydrogen. However, whether or not these three chemical products are really desired products or waste is decided not by the stoichiometry or scientific laws, but solely by the setting of preferences in the technosphere [14].

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This is made clear especially by looking at the example of chlorine-alkali electrolysis. When it was invented a hundred years ago, the process served primarily to produce caustic soda. The large quantities of chlorine generated could not be used and substantial surpluses resulted [15]. Chlorine was an undesirable by-product, in the worst case even a harmful emission. Allocation here is superfluous. Despite the joint process there is practically only one product – the caustic soda.

Faber et al. have pointed out that by-products from production processes are frequently initially only given off to the environment as wastes or emissions [16]. It was only in the course of time that they developed to become products for which a demand exists. The classification into product and waste categories cannot be decided on scientific grounds.

In the case of chlorine a substantial demand for the by-product only developed with chlorine chemistry. From time to time chlorine-alkali electrolysis was now only carried out because of the chlorine. Hydrogen was added, which could for instance be burned, serving as a substituted for other energy sources and thus reducing the process costs.

With such a reaction it makes no sense to allocate on the basis of masses or stoichiometry. The (variable) preference setting must be taken into account. What is product and what is waste? What product is desired and to what extent? Why is the process being carried out in the first place? Which substance is the driver of the process, and ultimately also the cost unit of output which after all finances the progress?

In our society the preference for products is mapped chiefly via the market price. Thus for example the price for chlorine was initially low, and that for caustic soda was high. Later the price ratio was inverted and has changed a few times over the years [14]. However, the disadvantage of the market price is that it does not always express the ecological truth, in other words external costs are not taken into account. Furthermore, it can fluctuate strongly within short periods, which renders calculations difficult. This was also the key argument in [2] for not wanting to use value-adding for allocation: "*It has the great disadvantage that it depends on the respective economic conditions and is therefore not a constant factor.*" The result of an LCA would then change equally quickly. However, a mass-related or stoichiometric distribution would feign causality and stable results of the LCA which do not exist in this form.

4. Optimising

The impression remains that selection of the allocation rule, in this case allocation on the basis of market prices, is arbitrary – and thus ultimately so are the results.

However, it can be shown that it is not the choice of the allocation rule, but instead the premises and objectives that are arbitrary, but these involve clear allocation rules. However, if objectives by subjects (in other words by individuals or society) play a role, then natural sciences can no longer make any statements. This is then the field for social and economic sciences.

The economist Gümbel [17] set out along an interesting path in the area of classical cost accounting. He posed the question of attribution of common costs to the cost unit, in other words the products, and sought a method free of arbitrariness, which has so far always been considered a hopeless venture. His suggestion assumes a benefit function U for all products i of the company. y_i are the various quantities of the product sales. U as a benefit function can be interpreted in monetary terms and contains the profit function; however, non-monetary components can also be taken into account, for example the prestige of the company. This benefit function will be maximised:

$$\text{Max } U = f(y_1, \ldots y_N) \qquad (1)$$

Such optimising must be carried out observing the restriction U = K + G, where K represents the costs and G the profit.

Gümpel was able to derive a formula from this optimisation problem in which the product prices required only depend on the quantity sold and the marginal benefit values of the products. However, this is ultimately the viability principle, as the calculation of the costs depends on the benefit assessment of the products. The allocation is no longer arbitrary, but is defined by the optimising goal. However, it is necessary to make an appropriate selection of a benefit function U. And this is precisely what is arbitrary here. The benefit can be different depending on the target group, e.g. in-company or consumer.

This approach can also be transferred to allocating an ecological burden, for instance the burden of emissions [14]. The by-products generate a benefit U, which can be measured. This benefit is to be maximised. For the sake of simplicity we assume two products, whereby the y_i describe the known product quantities. The benefit U depends on the product quantities. Furthermore, the total emissions E of the process are known. This now results in the constraint in which the two e_i describe the ratio of distribution, in other words the allocation rule.

$$\text{Max } U = f(y_1, y_2) \qquad E = e_1 \cdot y_1 + e_2 \cdot y_2 \qquad (2)$$

The e_i are unknown, but result from the optimising problem that can be solved using the Lagrange function in standard form:

$$L = U\left(y_1, y_2\right) + \lambda \cdot (E - e_1 \cdot y_1 - e_2 \cdot y_2) \qquad (3)$$

$$\frac{\partial L}{\partial y_1} = \frac{\partial U}{\partial y_1} - \lambda \cdot e_1 = 0 \qquad (4)$$

$$\frac{\partial L}{\partial y_2} = \frac{\partial U}{\partial y_2} - \lambda \cdot e_2 = 0 \qquad (5)$$

$$\frac{\partial L}{\partial \lambda} = E - e_1 \cdot y_1 - e_2 \cdot y_2 = 0 \qquad (6)$$

The factors e_i and with them the allocation rule can be clearly calculated by solving the equation system (3)-(6). Accordingly they result from the requirement that the benefit of the system be maximised. As shown in [14], the benefit can also include ecological aspects alongside economic factors.

5. Game theory approach

The allocation can also be derived from game theory. Here the game winnings or the costs, which are divided up between the players, are at stake. Assuming certain game rules and axioms, distinct allocations can be derived. They are used in the world of business and industry for many practical questions, not least in the field of overheads distribution [18].

A joint production process can be understood as a cooperative game between a number of players who are each responsible for their products and their benefit. The basic idea is that joint production, in other words cooperation, is more gainful than if the products are manufactured individually (and the other outputs possibly treated as wastes).

The Shapley value has evolved from these cooperative games [19]. In 1953, Shapley introduced it to determine the value for each player of participating in a cooperative game. He proceeded axiomatically. He demanded symmetry, so that the identity of the player and hence the sequence of joining the game is irrelevant. He also demands efficiency, i.e. the proceeds, game winnings or costs are distributed completely. Finally, he demands additivity. This means that if two games are combined, then each player receives the payoff corresponding to the total of his payoff from the two individual games. The value that satisfies these conditions is the Shapley value. It can be interpreted as a recommendation on how the payoffs or contributions of the game are divided between the individual players. This is demonstrated below with an example.

6. Milk run as an example

Goods transport is a classic allocation problem. In LCA practice this is generally solved by assuming average values for the truck payload and relating the costs of fuel consumption or the CO_2 emissions to the transport performance (in t-km) [13]. However, the allocation becomes distinctly more difficult at the latest when it is necessary to include empty runs for the return trips or on round trips (known as the milk run).

Looking at a milk run by way of example (Fig. 1), starting from the initial point (below), three (N=3) different destinations A, B and C are to be called at in order to deliver three consignments a, b and c. The goods could be delivered individually, or in different combinations of two, or in one combination of three – the actual milk run. This corresponds to coalitions in the game.

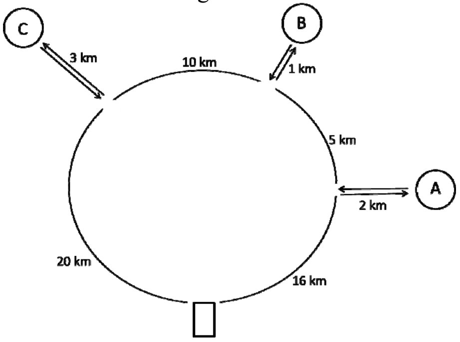

Fig.1: Milk run with three destinations to be called at.

There are various possibilities of allocation. For the sake of simplicity, first of all only the trips are divided up between the three cargoes, which each weigh the same. The payload of the vehicle is assumed to be constant.

The destinations can either be driven to individually. In this case trips of 36, 44 or 46 km are necessary. Or the trips can all be connected. In a "threesome coalition" the trip or "cost function" K = 63 km. But how are the "costs" of 63 km divided up between the three sets of goods? It is possible to be guided by what extra costs $\bar{K}(i)$ would be incurred if a coalition already exists and a further cargo is added.

$$\bar{K}(A) := K(N) - K(BC) = 4$$
$$\bar{K}(B) := K(N) - K(AC) = 2$$
$$\bar{K}(C) := K(N) - K(AB) = 15$$
$$\bar{K}(AB) := K(N) - K(C) = 17$$
$$\bar{K}(AC) := K(N) - K(B) = 19$$
$$\bar{K}(BC) := K(N) - K(A) = 27$$

Table 1: Possible coalitions and the respective cost function K in [km]

A	**B**	**C**	**AB BA**	**AC CA**	**BC CB**	**ABC CBA**
16	21	20	16	16	21	16
4	2	6	4	4	2	4
16	21	20	5	15	10	5
			2	6	6	2
			5	20	20	10
			16			6
						20
K(A)	**K(B)**	**K(C)**	**K(AB)**	**K(AC)**	**K(BC)**	**K(N)**
36	44	46	48	61	59	63

Then OV could be interpreted as a kind of overhead "costs", in other words those "costs" that cannot be clearly allocated to the individual cargoes, but instead are incurred jointly.

$$OV = K(N) - \sum_{i \in N}^{N} \bar{K}(i)$$
$$= 63 - 4 - 2 - 15 = 42$$

The simplest distribution of the "costs" would then be:

$$\bar{K}(i) + \frac{OV}{n}$$

The result can be seen in column 2 of Table 2. However, is it fair that the situation of b has improved by almost two thirds of its original value? Shouldn't a and c also be improved more strongly as a result of a coalition?

Ransmeier proposed the Alternative Cost Avoided approach in 1942 [20, 21]. The greater the difference between the stand-alone costs and the individual additional costs, the higher the burden for the beneficiary. The result is also shown in Table 2.

Table 2: Different "cost" distributions in [km]

	Stand-alone	**Simple OV**	**Ransmeier**	**Shapley**
a	36	18	16,8	16,5
b	44	16	18,8	19,5
c	46	29	27,4	27
	(126)	63	63	63

However, the allocation methods, including that of Ransmeier, all have disadvantages. Assuming that the subsection on the ring between B and C was not 10 km but instead 30 km long. Considered altogether, a threesome coalition would always be more interesting than the individual or double strategies. However, the distribution of costs according to Ransmeier would lead to C not having any interest in participating. C would drive "alone". The allocation approaches are therefore not stable – even though, viewed overall, the coalition would be advantageous.

Accordingly, distribution methods are necessary that are stable and feasible. Such cost allocations lie in the "core" of the game. In other words, this is a cost distribution in which there is no coalition of individual purposes that can come off better than the overall solution through separation or via sub-coalitions. The core is therefore the quantity of "stable" solutions.

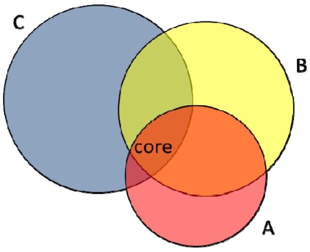

Fig.2: The cost shares of the three players and the possible savings as intersections

In graphic terms the core can be interpreted as the common intersection set of the three sub-quantities (Fig. 2). If it exists, then stable solutions are possible, in other words stable allocation rules. From among the many possible allocation rules that lie in the core, the Shapley value selects those embedded in efficiency and fairness of the solution. The value is based on the fact that the sequence of project coalitions is random. That is why all combinations of coalitions are played through, leading to the following formula:

$$y_i = \sum_{\substack{S \subset N \\ i \in S}} \frac{(s-1)!(n-s)!}{n!}[K(S) - K(S - \{i\}] \quad \forall i \in N$$

The results are also entered in Table 2. The solution is stable, efficient and "fair". This example could also be constructed with more reference to reality. Different weights of the goods to be transported, variable payload degrees of the transport vehicle and fuel consumption or CO_2 emission dependent on payload can be assumed.

Finally, the example can also be transferred generally to joint production. The various coalitions become possible as a consequence of this because the choice of what is product and what is waste is, in fact, not god-given or scientifically stipulated. For example alkali electrolysis could only be operated because of the demand for chlorine; or because of chlorine and caustic soda together, etc. This in turn results in coalitions with a characteristic function for electricity consumption or CO_2 emissions which can be used for the appropriate allocation rule.

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