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Technical Change and the Elasticity of Factor Substitution

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Abstract

This paper addresses the relationship between technical change and the elasticity of substitution between factors of production. It is shown how the elasticity within a CES production setting can change due to technical change. Technical change is interpreted in the spirit of horizontal differentiation as in many growth models. Cases for positive and negative returns to differentiation are analyzed which can be understood as progress or complexity congestions. It is shown how the elasticity changes due to technical choices for each of them.

Keyword: Elasticity of substitution, CES production function, Inequality

JEL-Classification: E23, O33, F41, J24
1 Introduction

Recently, there have been several attempts to introduce more flexibility into the well-known CES production function developed in Arrow et al. (1961). This interest in the CES is motivated by at least two considerations. First, the elasticity of substitution is a key parameter that shapes the production function. Numerous recent contributions analyze in detail the influence of this elasticity on growth related issues (e.g. Klump and de La Grandville 2000, Papageorgiou and Miyagiwa 2003, Papageorgiou and Saam 2008, Palivos and Karagiannis 2010, Wong and Yip 2010 or Irmen 2011). While a constant elasticity of substitution might be a reasonable local approximation, this elasticity might well be subject to changes over time or factor intensities. This was already conjectured by Arrow et al. (1961). This issue is of special importance in growth related analyses. This leads us directly two the second motivations. To account for such changes demands for more flexible production functions. In the CES context there are several approaches. Antony (2009a,b 2010) gives more flexible CES representations based on ad hoc normalization techniques. But the importance of the substitution elasticity has researchers also motivated to provide us with micro foundations that allow a CES function to be the solution to different optimization problems. Depending on the optimization problem, different values for the elasticity of substitution can arise. Contributions that take up this issue can be found in e.g. Acemoglu and Zilibotti (2001), Jones (2005), Nakamura and Nakamura (2008) and Nakamura (2010).

This paper provides a framework that links the two last approaches. It provides a micro foundation through the solution of an optimization problem. This is related to technical change and leads to a production function with a flexible structure for the elasticity of substitution, i.e. the resulting elasticity is a function of a parameter of the underlying optimization problem The production setting is richer parameterized and allows for more cases to be considered than has been done previously in the literature. This is done by specifying at the outset a quite standard CES production over differentiated input factors provided by two underlying primary factors. The degree of differentiation for these two factors is subject to the producers’ choice which links the present approach to the theory of technical change. After accounting for this endogenous choice, a CES production function in the two primary factors emerges with a different elasticity of substitution. The difference is related to what is known as the returns to differentiation or love of variety.

The above theory is developed using capital and labor as primary input factors. However, any two primary input factors could have been chosen. Additionally an application of the theory to inequality within a north-south model is provided. In this setting, the north solves the technology optimization problem which can lead to relative inefficiencies for the southern economy that has to adopt that technology. Applied to the primary factors capital and labor, this can lead to increasing inequality if the northern technology setter is to use a more capital intense mode of production.

The next section gives a review of the related literature. Section 3 develops the theoretical production model. In Section 4, this theory is applied to inequality problems between countries and skill groups. Finally, the last section concludes.

2 Related Literature

Antony (2009a) introduces the dual elasticity of substitution production function for the two input factors capital and labor given by equation (1). It is theoretically derived using the normalization technique in de La Grandville (1989).

\[ y = \left[ \alpha + \left( 1 - \alpha \right) k \right]^{\frac{1}{1-\beta}} \left[ \alpha + \left( 1 - \alpha \right) k_0 \left( \frac{k}{k_0} \right)^{\beta} \right]^{\frac{1}{\beta}}, \tag{1} \]

where \( k \) is the capital intensity, i.e. capital input per worker, and \( y \) is output per worker. \( 0 < \alpha < 1 \) is a share parameter. From (1) it becomes clear that the elasticity of substitution is \( \sigma = \frac{1}{1-\beta} \) for \( k \neq k_0 \) but \( \sigma = \frac{1}{1-\gamma} \) for \( k = k_0 \). This implies that labor and capital can be differently substituted for, depending which value \( k \) takes. This specification can be given economic meaning if e.g. \( k_0 \) is interpreted as a reference value for the capital intensity that set is by a technology leader. This production function has been applied in a theoretical setting in Antony (2009b) to elaborate on the relationship between capital deepening and foreign direct investments. Antony (2010) extends (1) to a changing
elasticity of substitution production function where the elasticity of substitution can take different values over different ranges for $k$ and calibrates it using cross country data.

While the above function provides greater flexibility with respect to the elasticity of substitution, it fails at providing an argument why a production function takes such a specific formulation. There is thus, no intuition provided why the production possibilities in an economy might take this form. At best, one can give empirical evidence that supports the choice for such a functional form.

A different strand of the literature does not suffer from this last shortcoming. Several contributions provide micro foundations that give support to production functions belonging to the CES family. Houthakker (1955-1956) shows how to aggregate out a Cobb-Douglas production from underlying Leontief production functions when their coefficients are jointly Pareto distributed. Jones (2005), partially building on this finding, shows that a global Cobb-Douglas production function can arise from a general constant returns to scale production function. This result holds after input factors have been chosen in a profit maximizing way in response to realizations of factor specific augmentation coefficients. The factor augmenting terms have, however, to follow a joint Pareto distribution.

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Leon-Ledesma and Satchi (2011) develop a technology choice model similar to the one that will be developed below. They show how through a parameter choice in a specific CES production function with an elasticity of substitution below unity a Cobb-Douglas reduced form production function can be established. However, the problems solved in Leon-Ledesma and Satchi (2011) and in this contribution are non-nested.

Acemoglu and Zilibotti (2001) demonstrate how a Cobb-Douglas production function after endogenous technology adjustment leads to a CES production with a higher than unity substitution elasticity. Their approach is closely linked to the one in the present contribution. In their specification two primary input factors are differentiated over a unit interval of intermediate inputs. Each input uses just one of the primary factors. Productivity of the intermediate inputs depends on their position in the interval through a specific functional form. Choosing which primary factor is used for each intermediate input in a profit maximizing way results in a CES production function with an elasticity of exactly 2 in the two primary factors. Nakamura (2010) and Nakamura and Nakamura (2008) extend this approach by using a more general functional form for the intermediates’ productivities. Doing so, they are able to show how a general CES production with an elasticity above unity can arise from an underlying Cobb-Douglas technology.

The following section shows how ideas from this micro founded literature can be extended to model an endogenous change in production from one CES to another CES function. Additionally, we will elaborate on the economic intuition behind such a change. Later on we will also show how a functions as (1) can endogenously arise and what economic intuition stands behind it.

3 The Model

We consider a production function defined over differentiated input factors that are obtained from either of two primary input factors. This function is of the type considered in Ethier (1982) and reads as

$$Y = \left( \alpha \int_0^1 l_i^\rho \, di + (1 - \alpha)(1 - J)^\rho \int_0^1 k_i^\rho \, di \right)^{\frac{1}{\rho}}. \quad (2)$$

There is a mass of 1 of differentiated input factors that can either be obtained from the primary factor $L$ or $K$ i.e. $l_i = \frac{i}{j}$ and $k_j = \frac{k}{1-j}$ as the varieties are symmetric. $J$ is the mass of input factors using $L$ as primary factor while $1 - J$ is using factor $K$. For concreteness, one can think of labor and capital in place of $L$ and $K$. The specification (2) allows for an extra parameter reflecting the returns to differentiation or the love of variety. As in Ethier (1982) this effect is captured by the parameter $\nu$. Note that for $\nu = \frac{1-\rho}{\rho}$, we would arrive at the returns to differentiation implied by a standard Dixit-Stiglitz (1977) index. $0 < \nu < 1$ is a scale parameter reflecting the importance of the two primary input factors.

Substituting $l_i = \frac{i}{j}$ and $k_j = \frac{k}{1-j}$ in (2) yields

$$Y = (\alpha J^\nu l^\rho + (1 - \alpha)(1 - J)^\nu K^\rho)^{\frac{1}{\rho}}, \quad (3)$$

which is a CES production function in labor and capital. The elasticity of substitution between labor an capital is $\sigma = \frac{1}{1-\rho}$ given $\rho$. 
As said before, a choice related to technology is introduced into the model similar as in Acemoglu and Zilibotti (2001), Nakamura and Nakamura (2008) or Nakamura (2010). The mass $1$ of differentiated inputs is split at $0 < J < 1$ into a domain $J$ operated by labor and a domain $1 - J$ operated by capital. A decreasing $J$ could be interpreted as increasing mechanization of production where more tasks are performed by capital as e.g. in Nakamura and Nakamura (2008). The choice of $J$ in this context is less related to technical progress than to technical change. I.e. changing $J$ could be associated with a change in the way of production given the state of the art of technology. It might be possible to perform a certain task in production by considerable man power or by making use of a specialized machinery equipment. It is the choice of the producer how to design the production process.

It is this choice to which we turn now. It seems natural to formulate an optimization problem that gives a solution for $J$. Using the production function (2), results in the profit function

$$\Pi = Y - \int_0^J w l_i \, di - \int_J^1 r k_i \, di,$$

where $w$ is the wage and $r$ is the interest rate given exogenously to the optimization problem. The problem $\max \Pi$ gives the first order condition

$$\frac{wL}{J} = \frac{rK}{1-J}.$$

There is an obvious analogy with the results found in the literature on directed technical change (see e.g. Acemoglu 1998). The shares of the differentiated input factors in production are equalized when condition (5) is met. This first order condition characterizes a profit maximum if $v(p - 1) < 0$. If $v(p - 1) > 0$ is true, (5) does not characterize a profit maximum but rather a minimum. In such a case it would be optimal to either chose $J = 1$ or $J = 0$ depending on $w$ and $r$, and production would be entirely done by labor or capital alone. In the remainder we will deal primarily with interior solutions as these are the most interesting ones.

If the operator of the production function (2) is entirely maximizing profits, she will employ differentiated input factors to the extend that their marginal products equal their price

$$w = \alpha Y^{1-p} (1-j)^{1-p} L^{-1},$$

$$r = (1-\alpha) Y^{1-p} (1-j)^{1-p} K^{-1}.$$

This gives

$$\frac{w}{r} = \frac{\alpha Y^{1-p} (1-j)^{1-p} L^{-1}}{(1-\alpha) Y^{1-p} (1-j)^{1-p} K^{-1}} = \frac{\alpha Y^{1-p} (1-j)^{1-p}}{(1-\alpha) Y^{1-p} (1-j)^{1-p}}$$

which together with the first order condition for the optimal $J$ (5) gives

$$\frac{1-J}{J} = \left(\frac{1-\alpha}{\alpha}\right) \frac{1}{k_{1-p}} \left(\frac{L}{K}\right)^{1-p}.$$

or equivalently

$$J = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right) \frac{1}{k_{1-p}} \left(\frac{L}{K}\right)^{1-p}}.$$

$$1 - J = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right) \frac{1}{k_{1-p}} \left(\frac{L}{K}\right)^{1-p}}.$$

$^1$One could think of a unit mass of atomistic and symmetric firms each operating a production function as (2). In such a perfectly competitive setting, $K$ and $L$ would equal the aggregate supply of capital and labor. The wage and interest rate would be given exogenously to each firm.

$^2$See the Appendix for details.
Using this result in the production function (3) yields the reduced form production function after the endogenous adjustment of the technical choice variable $f$ as

$$
Y = \left[ \frac{1}{\alpha-\nu \gamma} \left( L^{\frac{1}{\rho}} + \frac{1}{\alpha} K^{\frac{1}{1-\nu}} \right)^{\frac{1-\nu \gamma}{\rho}} \right] \text{(11)}
$$

With this representation, the elasticity of substitution is now $\tilde{\sigma} = \frac{1-\nu \gamma}{\frac{1}{\rho} - \frac{1}{\rho}}$.

At this point it is interesting to analyze $\tilde{\sigma}$ a bit more in detail. For this we have to distinguish four cases\(^3\)

**Case 1 $\nu > 0, \rho > 0$:**
In this case we observe three different sub-cases, depending on the parameter values for $\nu$ and $\rho$:
- if $0 < \nu < \frac{1}{\rho} - 1$, $\tilde{\sigma} > \sigma > 1$,
- if $\frac{1}{\rho} - 1 < \nu < \frac{1}{\rho}$, $\tilde{\sigma} < 0$,
- if $\nu > \frac{1}{\rho}$, no interior solution.

**Case 2 $\nu > 0, \rho < 0$:**
Interior solution with $1 > \tilde{\sigma} > \sigma$.

**Case 3 $\nu < 0, \rho > 0$:**
No interior solution.

**Case 4 $\nu < 0, \rho < 0$:**
In this case we observe again three different sub-cases, depending on the parameter values for $\nu$ and $\rho$
- if $0 > \frac{1}{\rho}$, no interior solution,
- if $\frac{1}{\rho} > \nu > \frac{1}{\rho} - 1$, $\tilde{\sigma} < 0$,
- if $\nu < \frac{1}{\rho} - 1$, $\tilde{\sigma} > 1$.

\(^3\)See the Appendix at the end of the paper for details.
Figure 1 displays the parameter space for \( \rho \) and \( \nu \) and the solutions from the profit optimization problem. In all conventional cases \( \bar{\sigma} > \sigma \) holds. By conventional we mean \( \bar{\sigma} > 0 \). This is not surprising and is nothing else than the implication of the LeChatelier principle, if others factors are allowed to adjust, demand curves become more elastic. The elasticity of the factor demand curves is, of course, determined by the elasticity of substitution. Other factors are in this case \( J \). \( J \) determines how many input varieties are using labor or capital. Additionally, case 4 with a substantial negative \( \nu \) deserves extra attention. In this case, the elasticity of substitution before adjusting \( J \) is below unity while after adjustment it is above unity. Before adjustment, labor and capital are both necessary production factors (for \( \sigma \)). After adjustment, production is possible with just one of them (regardless what value of \( J \) has been chosen). This property is remarkable if one looks at the production setting from a different perspective. In this setting, the elasticity of substitution can be changed through technical adjustment. There is, thus, a link to the literature on variable elasticity of substitution (VES) production functions where the elasticity is changing depending on input choices. VES functions have the property that although the elasticity can change it is restricted. In all known functions the elasticity is either always smaller or larger than unity. A change from one region to the other can not be modelled (see also Antony 2010 on this). The endogenous change of the elasticity in the present setting is thus an exceptional property.

There are also unconventional cases in this setting. In case 1 and 4 above there are parameter constellations where \( \bar{\sigma} \) can be negative. This is true for intermediate values for the returns to differentiation. At first sight this seems to be rather odd cases as a negative elasticity is usually ruled out in conventional production functions. At this point, it should be mentioned that (11) is a reduced form rather than an original production function. There are several reduced form production functions in the literature that also possess a negative elasticity of substitution. We return to these cases in the section on applications. At this point we just want to note that in the reduced form (11) with \( \bar{\sigma} < 0 \), \( L \) and \( K \) are \( q \)-substitutes (Sato 1975), i.e. the marginal product of one factor decreases as the input of the other increases.

It is also interesting to look at a relevant special case when \( \nu = \frac{1 - \bar{\sigma}}{\rho} \). This is a relevant case since this delivers the conventional Dixit-Stiglitz (1977) index. In this case, the new elasticity of substitution \( \bar{\sigma} \) becomes infinitively large and the reduced form production function is linear. In case \( \nu = 0 \) it turns, unsurprisingly, out that \( \bar{\sigma} = \sigma \), i.e. technical change through the choice of \( J \) has no influence on the elasticity of substitution. A final special case, although less interesting, is \( \rho = 0 \) or equivalently \( \sigma = 1 \). In this case also \( \bar{\sigma} = 1 \) prevails.

We also have to comment on the economic interpretation of \( \nu > 0 \) and \( \nu < 0 \). Usually, the returns to differentiation or love of variety are tied to growth related issues. Often growth is modelled through an increase in the “number” of intermediate inputs. In such a case \( \nu > 0 \) is a necessary and plausi-
ble assumption while \( \psi < 0 \) would lead to negative growth not intended by these models. However, \( \psi < 0 \) has an economically meaningful interpretation if one thinks of complexity of production as harmful. In such a setting, an increase in the number of tasks to be done by one of the primary factors would reduce the productivity of this factor. This would be true if there are e.g. negative coordination externalities leading to congestions. As increasing complexity due to higher differentiation of primary input factor might well be a real world phenomenon, we regard this as a relevant case.

Finally, we have to discuss the issue of normalization. It is by now common practice to normalize CES production functions when drawing inference upon the elasticity of substitution (de La Grandville 1989, Klump and de La Grandville 2000). This is because the share parameter in a conventional CES function depends on the elasticity of substitution. A change in the elasticity induces a change in production also through this parameter. This is, however, not what we are doing here. We analyze just the influence of \( \sigma \) on \( \vartheta \) for which normalization is not important since the share parameter is not involved in this. If one were to conduct an analysis involving also the share parameter, normalization would become a relevant issue.

4 Application within a North-South Model

This section provides an economically meaningful foundation in terms of the above production setting for the functional specification chosen in Antony (2009a) discussed in the literature review above. We consider a world that consists for simplicity of just two countries, a northern country that sets worldwide technology standards and a southern country that is only able to adopt these standards to access world markets. Both countries use the technology given by the production function (3) above. The difference between the countries is that the northern country sets \( f \). In the following, variables are indexed by \( n \) and \( s \) denoting the north and the south respectively. Both countries sell their production on an integrated world goods market at the same price.

Using the intensive form, i.e. per capita variables, \( j \) is determined through the capital intensity in the northern country \( k_n = \frac{Kn}{l_n} \). This gives

\[
\begin{align*}
1 & = \frac{1}{1 + \frac{(1 - \alpha)^{1-\psi}}{\alpha} k_n^{1-\psi} \frac{\vartheta}{\varepsilon}} \quad (12) \\
1 - j & = \frac{\frac{(1 - \alpha)^{1-\psi} k_n^{1-\psi} \frac{\vartheta}{\varepsilon}}{1 + \frac{(1 - \alpha)^{1-\psi}}{\alpha} k_n^{1-\psi} \frac{\vartheta}{\varepsilon}}}{1} \quad (13)
\end{align*}
\]

Inserting this into the production function (3) gives production \( y_i \) for country \( i = n, s \) as

\[
y_i = \left[ \frac{1}{\alpha^{1-\psi}} + \frac{1}{(1 - \alpha)^{1-\psi} k_n^{1-\psi} \frac{\vartheta}{\varepsilon}} \right]^{-\vartheta} \left[ \frac{1}{\alpha^{1-\psi}} + \frac{1}{(1 - \alpha)^{1-\psi} k_n^{1-\psi} \frac{\vartheta}{\varepsilon}} \right]^{-\frac{1}{\vartheta}}
\]

(14)

Comparing this with (1) shows that both are of the same structure. Indeed, they are exactly identical with respect to the elasticity of substitution if one replaces \( \frac{1-\psi}{\vartheta} \) by \( \varepsilon \). The production function (1) is obtained in an ad hoc manner using a normalization approach. In the light of the preceding section, we have now an economically meaningful foundation that provides us with an intuition behind this function.

Antony (2009a) analyzes an interesting case regarding the elasticities \( \sigma \) and \( \vartheta \) based on empirical evidence.\(^4\) The analyzed scenario is characterized by \( \sigma < 1, \vartheta > 1 \) and different capital intensities in the north and south.\(^5\) In this case capital deepening in the northern country causes the capital intensity and per capita production in the southern country to fall giving rise to increased inequality between north and south.

\(^4\)This analysis is based on the results in Duffy and Papageorgiou (2000) who find an elasticity of substitution between labor and capital of above unity for developed countries and below unity for less developed ones. It should be noted that they use labor adjusted for human capital in their regression. Estimates using raw labor usually find also an elasticity below unity for developed economies (see e.g. León-Ledesma et al. 2010).

\(^5\)If both countries have access to a world capital market, different capital intensities can be the result of e.g. credit frictions in the south. These could be caused e.g. through monitoring costs (Townsend 1979, Gale and Hellwig 1985) that drive a wedge between the gross and net marginal product of capital in the south.
This scenario is, thus, implicitly based on the assumption that the returns of differentiation or love of variety are significantly negative (see case 4 above). As said before, such a situation could be interpreted as increasing differentiation of a primary input factor causes substantial negative effects through increasing complexity. As the north sets such technology standards, the south has to accept them in order to have access to the world’s goods market. However, in case of a more capital intense production in the north, it might suffer severe consequences that might widen inequality between the north and the south. Antony (2009a) provides a country panel analysis showing that this indeed happened in the last decades.

5 Conclusion

The present contribution builds a production set-up that endogenizes the elasticity of substitution in CES production functions. This endogeneity is induced through the introduction of a technology choice problem. In conventional cases, this choice leads to an elasticity that becomes larger. Unconventional cases with a negative elasticity in the reduced form production function are possible. The paper adds to the literature as it extends a recent strand in the literature on micro foundations for CES production functions. Existing approaches are at some stage always restricted by the need for a Cobb-Douglas production function. In the present setting a CES specification is possible for the original as well as the reduced form production function.

The second contribution is in providing a deeper understanding of the implications of production functions given the necessary assumptions in the micro foundations. One application within a north-south model has been given and its implications for inequality have been discussed. Given the production set-up in the present paper, we are able to obtain micro foundations that are linked to technology and complexity necessary to explain increasing inequality in capital intensity and production. Of course, there are other possible applications of the production theory given in the paper. One could think about wage inequality between high and low skilled workers. A non-exhausting list of contributions in the area would include the articles by Kiley (1998), Acemoglu (1998, 2002a) and Acemoglu and Zilibotti (2001). In general, these are endogenous growth models which seem to be suited very well for the production theory developed in this paper. Future research might consider the role of technical choice developed here for these models.

6 Literature


Sato, K., 1975, Production Functions and Aggregation, North-Holland, Amsterdam.


Appendix

Derivation of the first order condition (5):

\[
\frac{\partial \Pi}{\partial j} = \frac{1}{\rho} Y^{1-r} \left[ \frac{Y^r - (1 - \rho) Y^{r-1-\rho}}{1 - j} \right] - \left[ \frac{Y^r - (1 - \rho)}{1 - j} \right] \left( \frac{1}{1 - j} \right)^{1-\rho} \frac{K}{1 - j} - \omega l_j + r k_j
\]

Due to symmetry we have \( l_j = \frac{1}{j} \) and \( k_i = \frac{K}{1 - j} \).

Using the first order conditions (6) and (7), we arrive at

\[
\frac{\partial \Pi}{\partial j} = \frac{1}{\rho} Y^{1-r} \left[ (v + \rho) j^r - (1 - j)^{r-1} \right] \left( \frac{K}{1 - j} \right)^{1-\rho} \frac{wL}{j} + \frac{rK}{1 - j}
\]

Setting this derivative equal to zero gives (5).

Condition for profit maximum:
rewriting (15) using (6) and (7) gives

\[
\frac{\partial \Pi}{\partial j} = \nu Y^{1-r} [j^{r-1} L^\rho - (1 - j)^{r-1} K^\rho]
\]

Building the second derivative with respect to \( j \) gives

\[
\frac{\partial^2 \Pi}{\partial j^2} = \nu (1 - \rho) Y^{r-\rho} \frac{\partial}{\partial j} \left[ j^{r-1} L^\rho - (1 - j)^{r-1} K^\rho \right]
+ \nu (1 - \rho - 1) Y^{1-r} [j^{r-2} L^\rho + (1 - j)^{r-2} K^\rho]
\]

(16)

If condition (5) is fulfilled, \( \frac{\partial}{\partial j} = 0 \). Therefore, if the first order condition (5) holds, (16) becomes

\[
\frac{\partial^2 \Pi}{\partial j^2} = \nu (1 - \rho - 1) Y^{1-r} [j^{r-2} L^\rho + (1 - j)^{r-2} K^\rho]
\]

which is negative only if \( \nu (1 - \rho - 1) = \nu (v - 1) \leq 0 \). Only in this case we observe an interior solution with a profit maximum.

Cases to consider

**Case 1: \( \nu > 0, \rho > 0 \):**
For an interior solution, \( 1 - \nu > 0 \) is required which implies \( \nu < \frac{1}{\rho} \). \( \overline{\sigma} = \frac{1 - \nu}{1 - \nu \rho - \rho} > 0 \) if \( \nu > 1 - \rho - 1 \). In this case

\[
\frac{1}{1 - \nu \rho - \rho} > \frac{1}{1 - \nu \rho} > \frac{1 - \nu}{1 - \nu \rho - \rho} > 0
\]
is required for \( \tilde{\sigma} > \sigma \) which is true whenever \( \tilde{\sigma} > 0 \). If, however, \( \frac{1}{\rho} \), we get the result for \( \tilde{\sigma} < 0 \).

**Case 2: \( v > 0, \rho < 0 \):**
For an interior solution, \( 1 - \nu \rho > 0 \) is required which is always true for \( \rho < 0 \). Also \( \tilde{\sigma} = \frac{1 - \nu \rho}{1 - \nu \rho - \rho} > 0 \) holds always. Additionally, \( \tilde{\sigma} < 1 \) is obviously satisfied. For \( \tilde{\sigma} > \sigma \), again \( \frac{1 - \nu \rho}{1 - \nu \rho - \rho} > 0 \) needs to be true which is always the case for \( v > 0 \) and \( \rho < 0 \).

**Case 3: \( v < 0, \rho > 0 \):**
For an interior solution, \( 1 - \nu \rho > 0 \) would be required which is impossible in this case.

**Case 4: \( v < 0, \rho < 0 \):**
For an interior solution, \( 1 - \nu \rho < 0 \) is required which implies \( v < \frac{1}{\rho} \). \( \tilde{\sigma} = \frac{1 - \nu \rho}{1 - \nu \rho - \rho} > 0 \) if \( v < \frac{1}{\rho} - 1 \). In this case

\[
\frac{\tilde{\sigma} > \sigma}{1 - \nu \rho - \rho} \quad \frac{1}{1 - \nu \rho} \quad \frac{1 - \rho}{(1 - \nu \rho)(1 - \rho)} \quad \frac{1 - \nu \rho - \rho}{1 - \nu \rho - \rho} > 1
\]

is required for \( \tilde{\sigma} > \sigma \) which is true whenever \( \tilde{\sigma} > 0 \). If, however, \( \frac{1}{\rho} \), we get the results \( \tilde{\sigma} < 0 \).
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